Progress on Transverse Impedance Modeling and Benchmarking with Space Charge

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Transverse Impedance Model

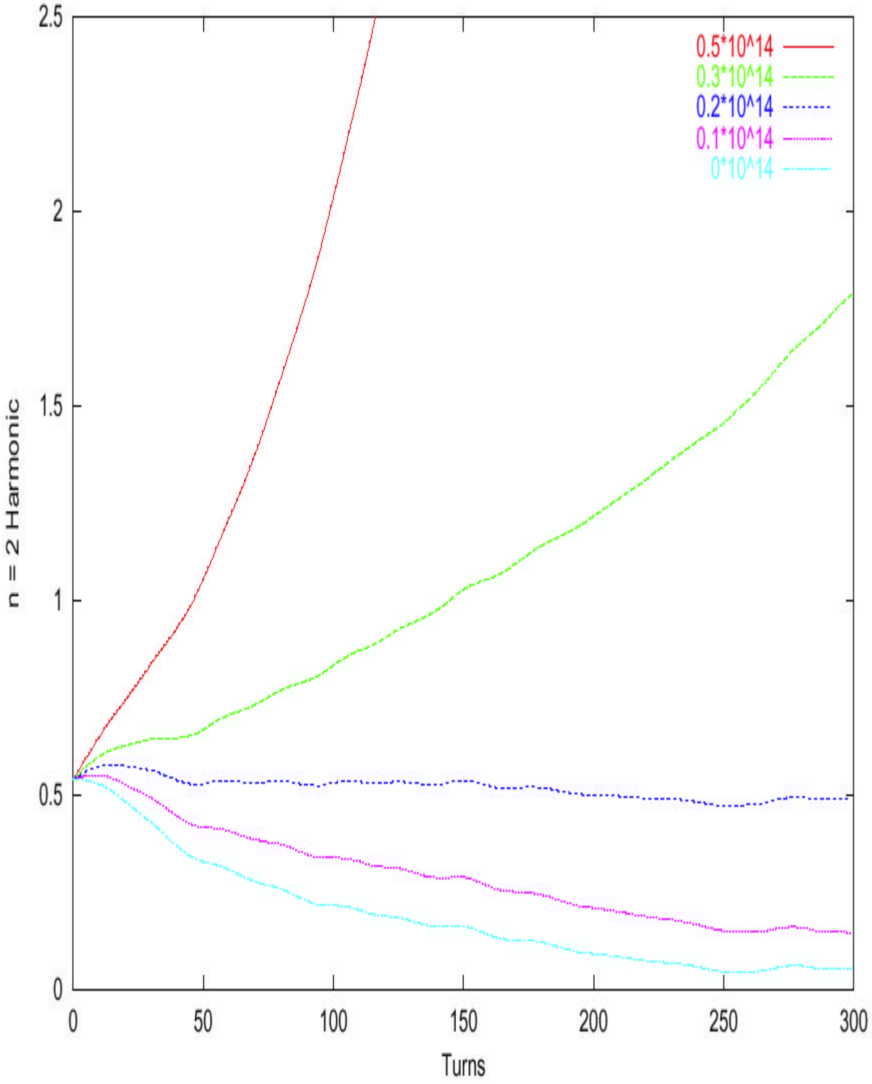
- Transverse impedance treated as localized node in ORBIT
  - Element length must be short compared to betatron oscillation wavelength
  - If physical impedance is not short, multiple impedance nodes are required
- Impedance representation
  - Fourier components at betatron sidebands of the ring frequency harmonics
  - Velocities less than light speed included in formulation
- Particle kicks
  - Convolution of beam current dipole moment with impedance
  - Current evaluation assumes dipole moment evolves from previous turn according to simple betatron oscillation

Transverse Impedance Studies: Benchmark

- Benchmark ORBIT with analytic calculation:
  - Straight uniform focusing lattice
  - Periodic length 40 m, tunes (1.10,1.05).
  - Longitudinally localized vertical impedance (b/a = 2, second harmonic, Z = $0.2*10^6$ Ohm in results shown below)
  - Coasting beam with
    - 1 mm 2^{nd} harmonic in y (slow wave);
    - Lorentz energy distribution (1 GeV, RMS width 1%, cutoff at 10%);
    - 10^{13} particles.
    - Use $2*10^5$ macroparticles.
    - Transverse distributions either "pencil beam" or KV with 30 pi mm-mr rms emittance.
- Analytic calculation for "pencil beam" with Vlasov equation and Landau damping.

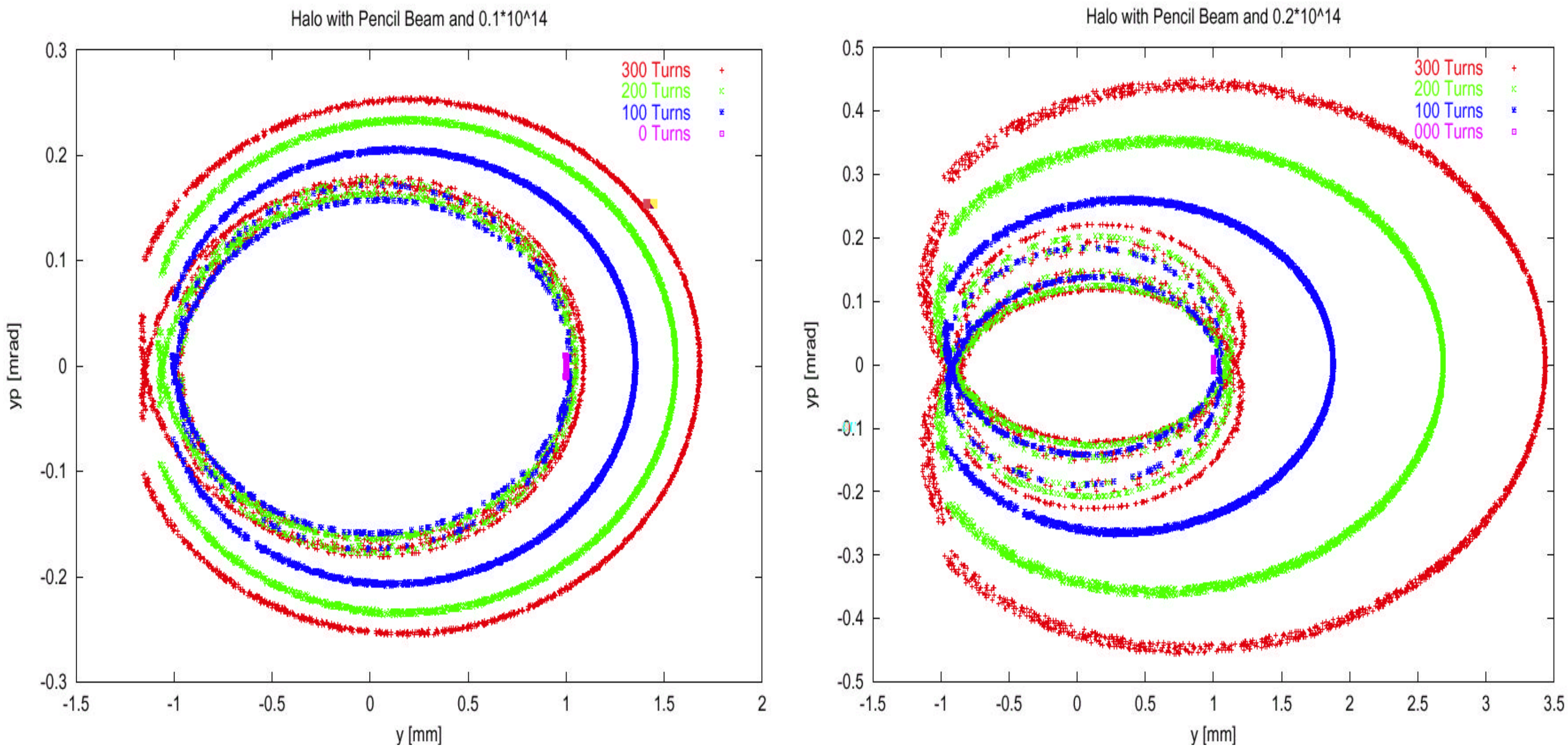
Transverse Impedance Without Space Charge: Find Instability Threshold

- Plotting the n = 2 harmonic vs turn number for several beam intensities places the threshold without space charge at about 0.2*10^{14} particles.



Distribution Evolution of Pencil Beam without Space Charge Forces

The evolution of the pencil beam distribution shows formation of halo even for stable cases. This has been benchmarked with analytic theory for another case, and will also be done for this case.

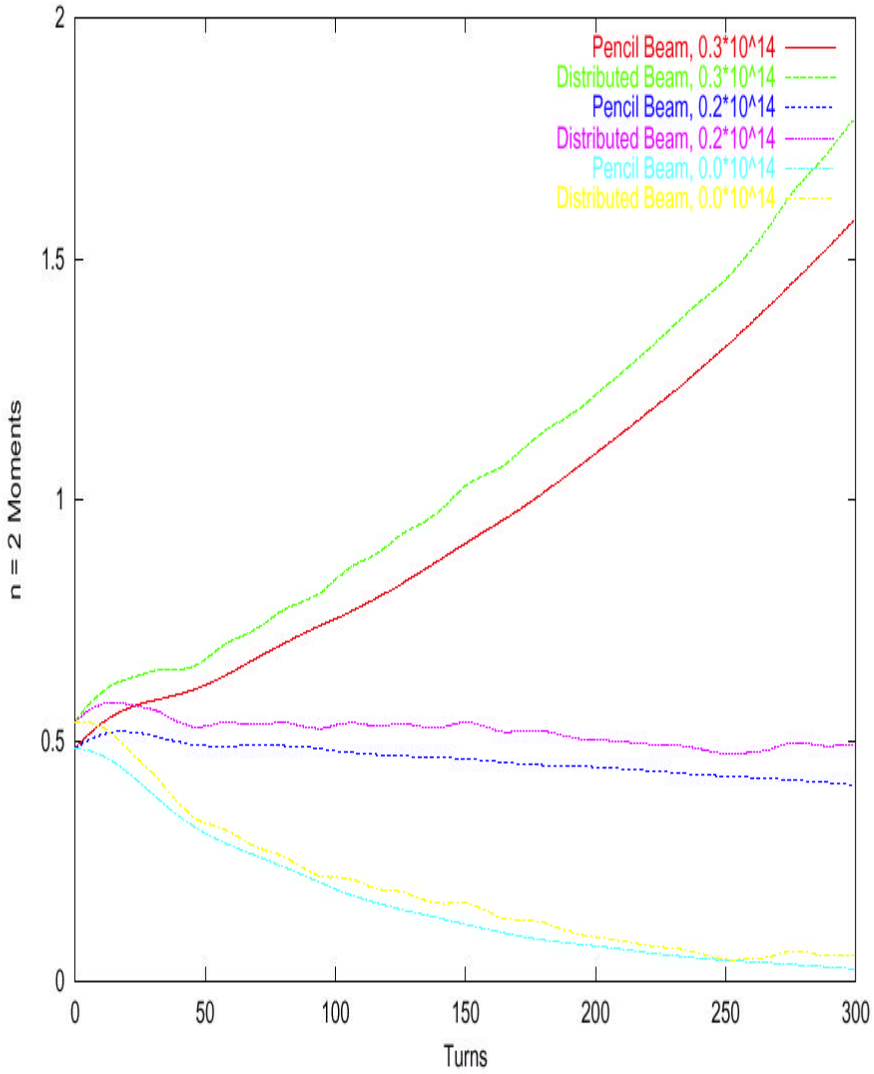


Need for 3D Space Charge Model

- For high intensity beams, transverse impedance and space charge forces must both be considered. For proper description of the beam dynamics, the space charge forces must be calculated using the full 3D beam distribution.
- The next slide shows that, when space charge forces are not included the beam evolution under transverse impedance forces is insensitive to the details of the transverse distribution, so long as the relevant beam moments are the same.
- 3D space charge forces are much more expensive to calculate than 2D models. The second slide following presents timings for a 2D FFT model, a 3D "brute force" force model, and a 3D FFT model. It is clear that, for scaling to finer grids, the 3D FFT model is far better than the "brute force" method.

Comparison of Pencil Beam and KV Distribution Evolution without Space Charge

- Pencil beam calculations are handy for detailed comparison of distribution functions with analytic theory, but for calculations with space charge, broader transverse distributions are required. Here we show that the evolution of the beam is insensitive to the transverse distribution when space charge forces are omitted.



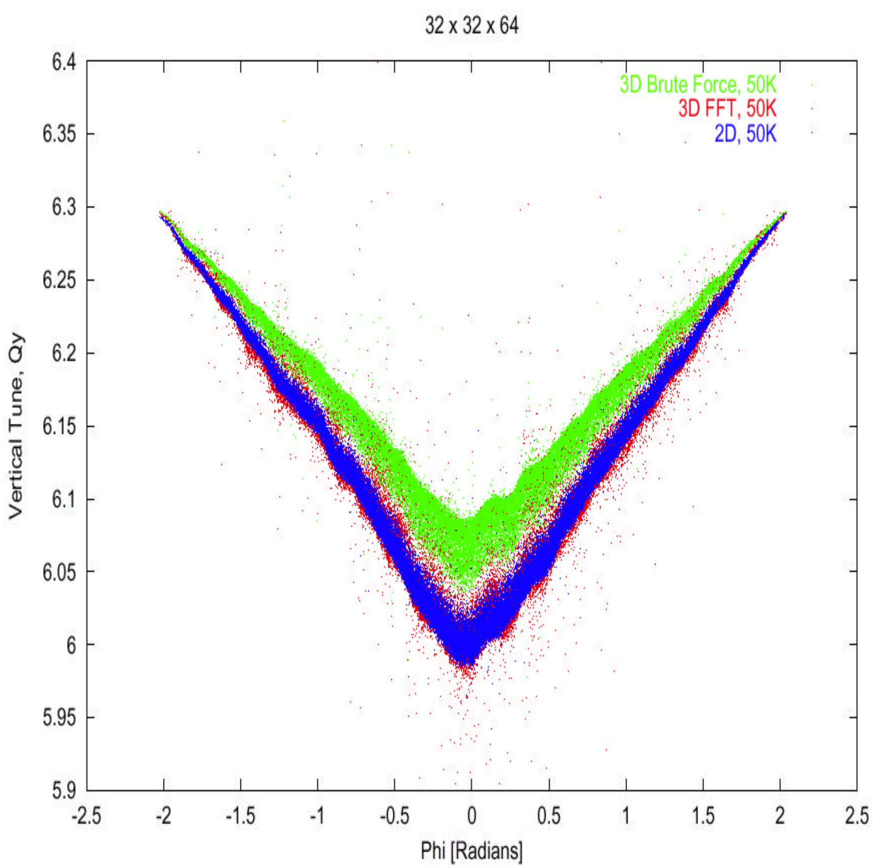
3D Space Charge - Timing

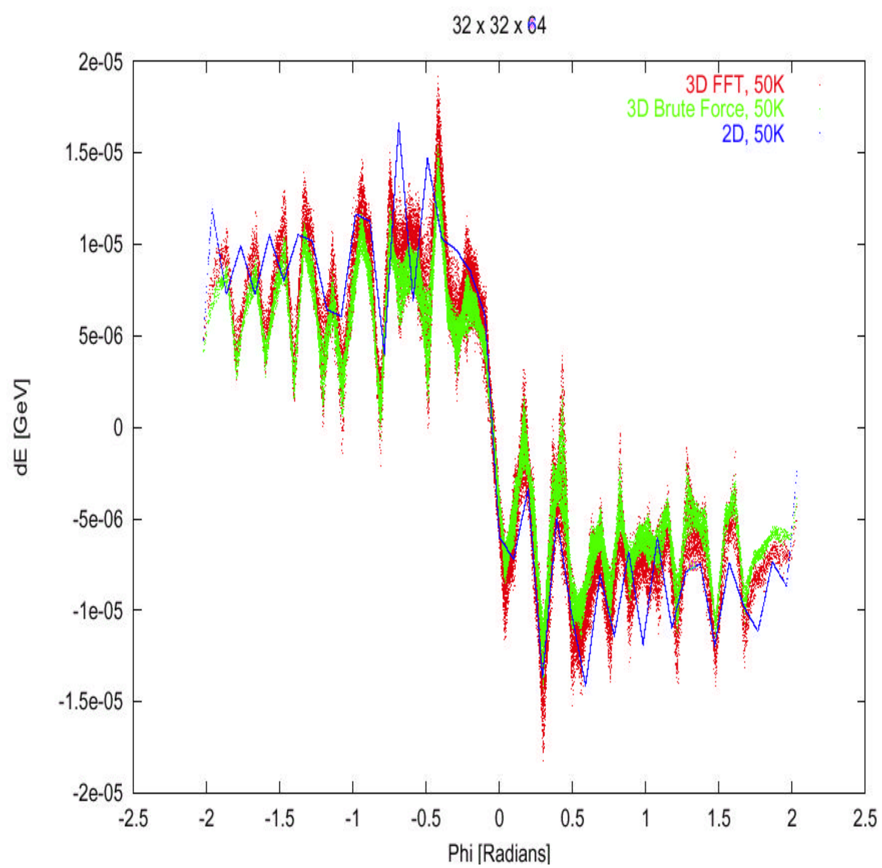
Run	Time(sec)	Time (mins)	Mins. Per Turn
2D, 50K, 32x32	453.2	7.55	1.51
3D-FFT, 50K, 32x32x64	2802.9	46.72	9.34
3D-BF, 50K, 16x16x64	4008.5	66.81	13.36
2D, 200K, 32x32	1837.9	30.63	6.13
3D-FFT, 200K, 32x32x64	5634.7	93.91	18.78
3D_BF, 200K, 16x16x64	6775.9	112.93	22.59
2D, 200K, 64x64	2076.9	69.22	13.84
3D-FFT, 200K, 64x64x64	7993.8	133.23	26.65
3D-BF, 200K, 32x32x64	57057.4	950.96	190.19

- Lattice: 248 Nodes

Numerical Convergence of Space Charge Model: Test for 2.5D Case (Triangular Beam)

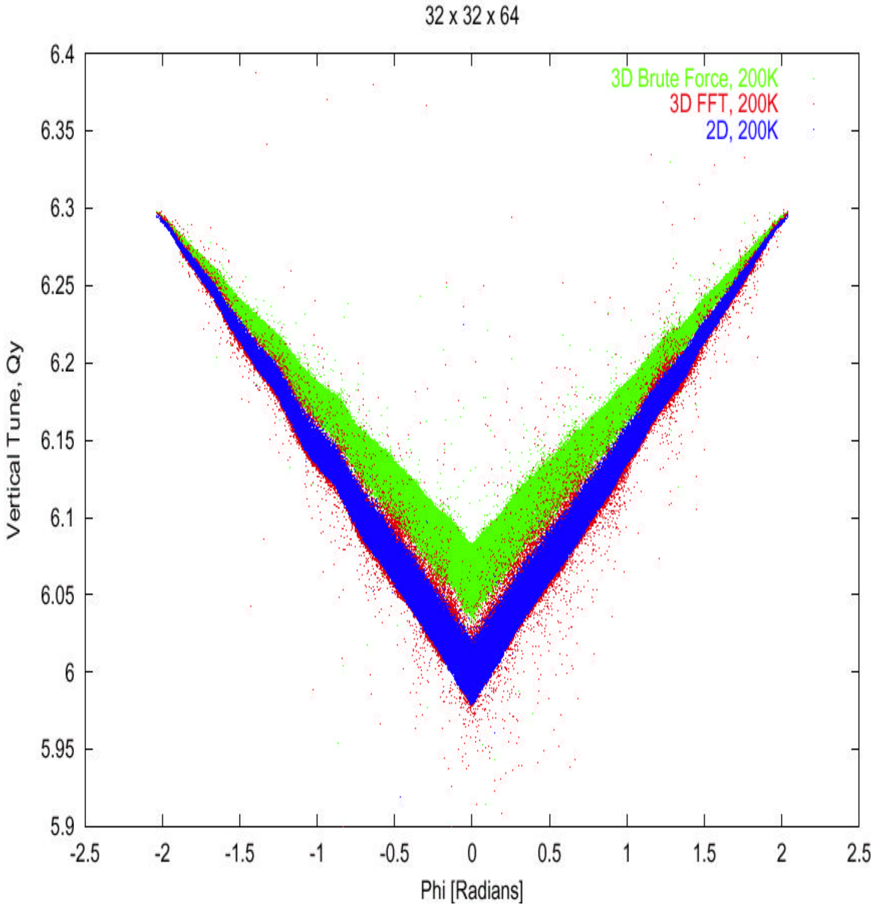
With 50000 particles and a 32x32x64 grid, the brute force tune shift disagrees with the 2D and 3D FFT tune shifts, which are in reasonable agreement. To a lesser extent, this also applies to the energy distributions. No impedance forces are present.

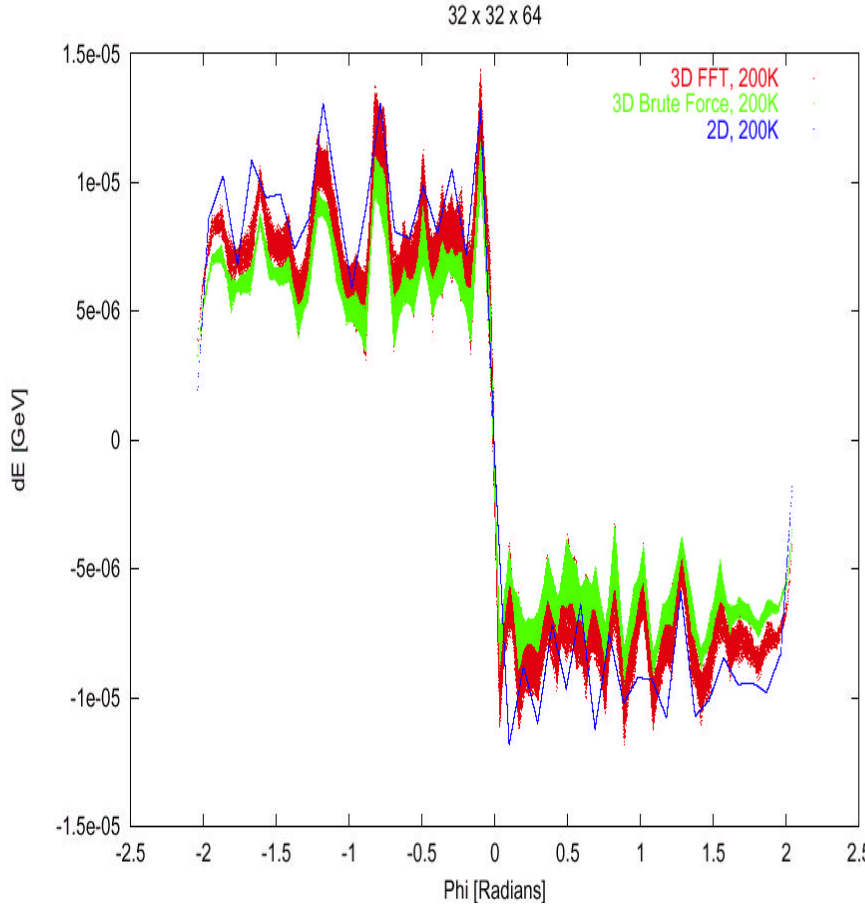




2.5D Case (Triangular Beam) Continued

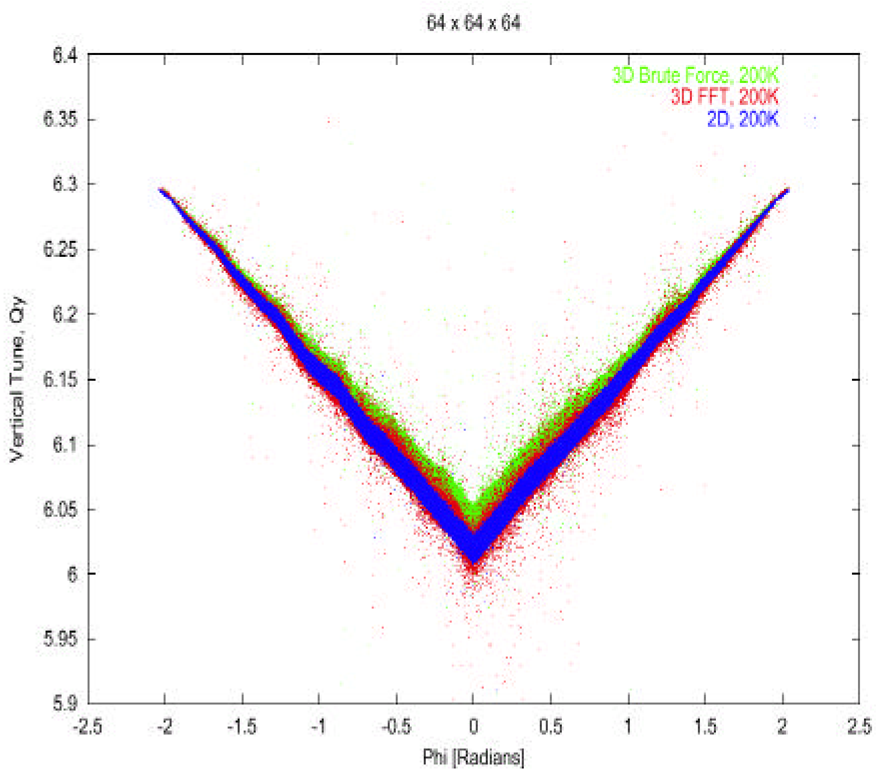
When the number of particles is increased to 200000, but the grid fixed at 32x32x64, the 3D brute force method continues to give different answers than the 2D and 3D FFT methods.

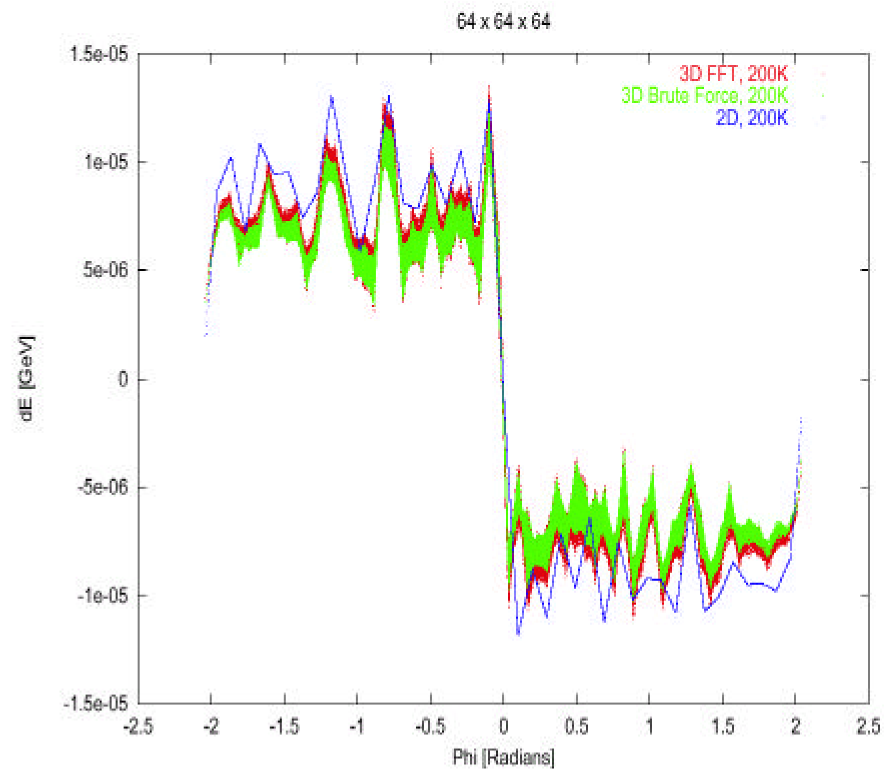




2.5D Case (Triangular Beam) Continued

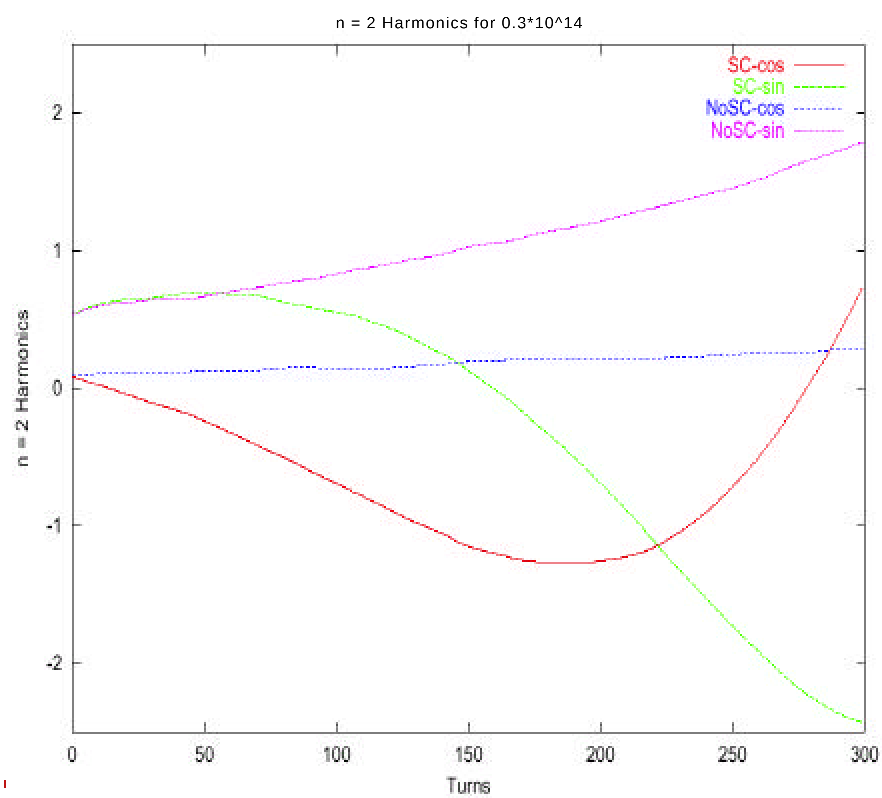
When the number of particles is increased to 200000 and the grid is increased to 32x32x64, the 3D brute force method agrees much better with the 2D and 3D FFT methods.

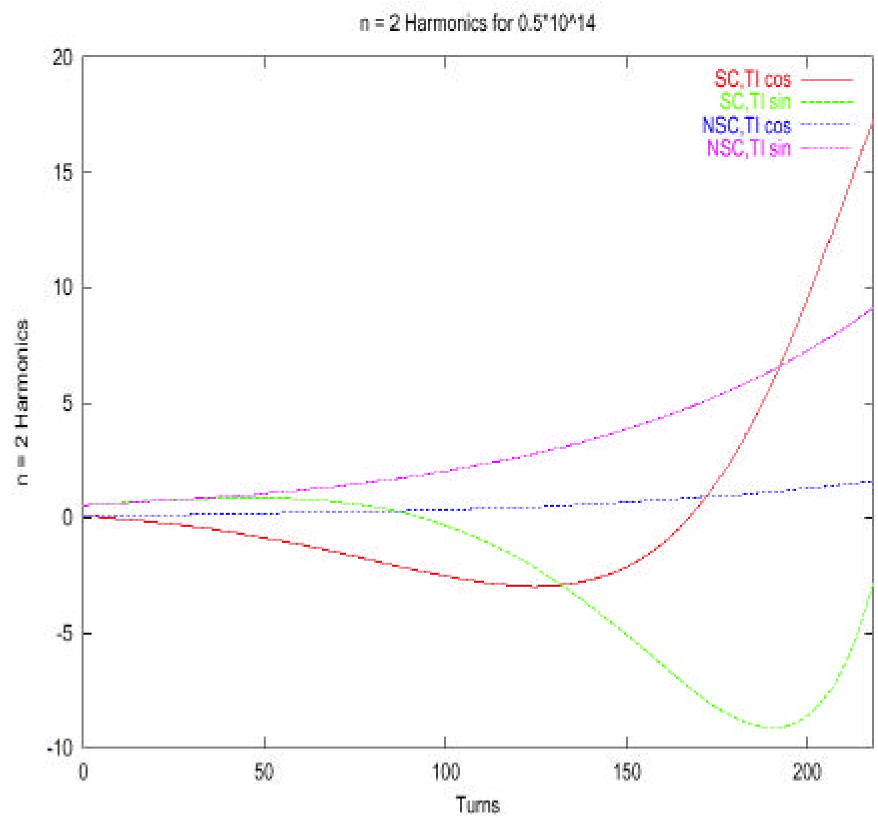




3D Space Charge and Transverse Impedance

For the transverse impedance benchmark case studied above, the addition of space charge appears to increase the growth rate and to cause the growing harmonics to oscillate, consistent with the space charge being an imaginary impedance.





Demonstration of the Need for 3D Space Charge and Impedance

The first figure shows the effect of the impedance term in the presence of the 3D space charge. With zero impedance, the beam harmonics quickly diminish due to Landau damping. The second figure shows the need for 3D space charge model in transverse impedance calculations. The results with the 2D model differ significantly.